Difference Between Two Minima Normalized Min-Sum Algorithm For The LDPC Decoder

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ABSTRACT

This study proposes the Difference Between Two Minima Normalized Min-Sum (DMNMS) algorithm for layered scheduling, which is suggested for implementing 5G New Radio Low-Density Parity-Check codes. The proposed algorithm aims to reduce memory space and enhance error correction performance. By applying modified factors to both variable nodes and check nodes processing, the DMNMS method goals to strengthen the protection ability for degree-1 variable nodes and compensate for the overestimation of extrinsic messages in Min-Sum-based algorithms. The suggested technique also used the difference between the first two minimum values and the first minimum among the input information, rather than compressing the check to variable messages to deliver the first two minima. Simulation data indicated that the proposed DMNMS algorithm performs better than the Normalized Min-Sum method, achieving up to 0.07 dB of decoding gain at a Bit Error Rate of 10^{-6} while using less memory.

KEYWORDS: 5G, Check node processing, Low-Density Parity-Check (LDPC), Min-Sum, Normalized Min-Sum (NMS)

1. Introduction

Error Control Coding (ECC) is widely used in many communication and memory systems to reduce the probability of errors (Huang et al., 2019). Several approaches in literature have been studied for designing and developing feasible coding schemes that can reduce data transmission and reading errors (Egilmez et al. 2019; Babar et al. 2018). Among different coding schemes, Low-Density Parity-Check (LDPC) codes stand out as a unique class of linear block codes that have recently gained popularity due to their adaptable error-correcting features (Abdessalem et al. 2020). LDPC codes that utilize sparse matrices offer performance that comes close to the Shannon limit (Richardson et al. 2001). They have low decoding complexity, low error leveling, and significant flexibility. The extremely adaptable structure of LDPC codes enables full parallel operations. High throughput is made possible by the low complexity of their hardware implementation. Additionally, LDPC codes exhibit exceptional error correction capabilities in unexpected circumstances and have the potential for high-speed decoding.

Several digital communication standards use the conventional LDPC codes, created by R. Gallager (Gallager 1962), which are randomly produced codes with high error correction capabilities. Numerous communication and storage systems, including Advanced Television Systems Committee (ATSC) (Ahn et al. 2018), Wi-Fi (IEEE 802.11n) (Usman and Mansour 2020), WiMAX (IEEE 802.16e) (Telagam et al. 2021), and DVB-S2 (Chen et al. 2018), have embraced LDPC codes throughout the last few decades. In particular, the 5G standards approved LDPC codes as the channel coding method for the enhanced mobile broadband (eMBB) scenario (Tran-Thi et al. 2023a; Cui et al. 2020). Nevertheless, a significant research issue remains: creating low-complexity, high-performance LDPC decoding algorithms and matching Very Large Scale Integration (VLSI) implementations for 5G New Radio (NR).

In general, decoding algorithms for LDPC codes can be divided into two major classes, called soft-decision (Roberts et al. 2019) and hard-decision (Chen et al. 2020) based techniques, depending on their error-correcting processes. Among decoding methods, message-passing techniques, such as the Belief Propagation (BP) algorithm, which iteratively exchanges messages along the edges between variable nodes (columns) and check nodes (rows), are typically used to decode LDPC codes. Nevertheless, the BP decoding has a significant memory overhead and computational complexity (Mansour and Shanbhag 2003). The Min-Sum (MS) approximation has been used to develop several efficient algorithms that provide a better balance between complexity and performance for the conventional regular or irregular LDPC codes (Declercq et al. 2014). Consequently, enhanced MS variants have been introduced (Chen et al. 2005), including multiple-dimensional modified MS decoding (Zhang et al. 2006; Kang et al. 2020), Adaptive MS (AMS) decoding (Wu et al. 2010; Le Trung et al. 2019; Cui et al. 2020), self-correction MS decoding (Boncalo et al. 2016), Offset MS (OMS) decoding, and Normalized MS (NMS) decoding (Declercq et al. 2014; Chen et al. 2005). Therefore, there is still room for improvement to address some problems of the use of hardware resources, coding gain enhancement, convergence speed, and throughput requirements. To further enhance the NMS algorithm decoding performance, a Difference between two Minima Normalized Min-Sum (DMNMS) approach is provided in this study.

The paper is structured as follows: basic details and a definition algorithm are given in Section 2. In Section 3, the findings and related discussions are provided. Section 4 presents conclusions.

2. Preliminaries

Since the 5G standard requires a peak downlink throughput of 20 Gbps (5G. 2017), the 5G NR LDPC decoder must balance energy consumption, area efficiency, and throughput in terms of hardware implementations. Compatibility across all code configurations must also be maintained, which is a difficult task. Additionally, the decoder area is already heavily occupied by the enormous amount of memory needed to support the maximum block length in 5G NR, so these big memories tend to lessen the impact of more complex algorithms on overall efficiency.

Using an approximation computation for check node update processing is one way to increase the efficiency of hardware consumption (MS algorithm). More specifically, only the first minimum value is computed among input variable-to-check messages rather than the first two minimum values. Hardware resources can be significantly reduced with this method. For example, a novel approach called single minimal Min-Sum (smMS) was suggested in (Darabiha *et al.* 2006; Angarita *et al.* 2014). To determine the second minimum, a scaling parameter must be added to the first minimum value. Although this approach appears straightforward, it produces an error floor too soon. In reality, nearly all LDPC applications use Quasi-Cyclic LDPC (QC-LDPC), a particular kind of LDPC code (Fossorier 2004). A structure like this enables high parallelism in decoder implementations, which is necessary to meet the high throughput requirements of contemporary communications. It is important to highlight the notable inconsistencies in the 5G QC-LDPC codes (Wu and Wang 2019; Tran-Thi *et al.* 2023a; Wang *et al.* 2020). To reduce memory usage, the authors in (Lee *et al.,* 2016) suggested an effective rearranged tiered scheduling system based on the check-node degree and the number of punctured edges. Nevertheless, more hardware resources are required. The authors in (Tran-Thi *et al.* 2023b) can reduce memory space under the greatly fluctuating Check Node degree of 5G LDPC by using a split storage strategy.

Based on the NMS method, a new decoding technique is developed in this study that considers both the hardware implementation complexity and the decoding performance of the LDPC decoder. The decoding efficiency is enhanced by applying two-dimensional (2D) scaling factors to the first two minima, as stated in (Cho *et al.* 2017). This paper proposed a suggested approach that calculates the 2D scaling factors to minimize hardware costs without compromising decoding performance. In comparison to modified versions of MS methods, these scaling factors are further optimized by utilizing the difference between the first two minimum values of the Check-To-Variable message. This results in an enhancement of coding gain while requiring less memory space.

LDPC codes are linear block codes with a block length of N and a dimension of K. Its parity-check matrix $\mathbf{H}$ consists of M rows and N columns, where each row corresponds to a check node (or parity-check bit), and each column corresponds to a variable node (or coded bit). The LDPC-coded bits are now modulated using Binary Phase-Shift Keying (BPSK) and sent over an Additive White Gaussian Noise (AWGN) channel. The codeword C can be represented by the vector $\boldsymbol{c} = (c_1, c_2, \cdots, c_N)$ of size N. Before transmission, $x_n = 2c_n - 1$ maps it to $\boldsymbol{x} = (x_1, x_2, \cdots, x_N)$. The received vector is obtained at the receiver $\boldsymbol{y} = (y_1, y_2, \cdots, y_N)$, where $y_n = x_n + v_n; n = 1,2, \ldots, N$ and v_n is the additive white Gaussian noise with zero mean and variance σ^2.

We denote Check Nodes (CNs) linked to the Variable Nodes (VNs) n are represented by the set $H(n)$. VNs linked to the CNs m are represented by the set $H(m)$. $H(n)\backslash m$ is the set of $H(n)$ with the CN m excluded. $H(m)\backslash n$ is the set of $H(m)$ with VN n excluded.

The following symbols, which are used throughout the study, relate to MS algorithms that operate on an LDPC code's Tanner graph (Declercq *et al.* 2014).

γ_n: a priori information of the variable node n,

$\tilde{\gamma}_n$: A-Posteriori (AP) information of the variable node n,

$\alpha_{m,n}$: the Variable-To-Check (VTC) message from n to m,

$\beta_{m,n}$: the Check-To-Variable (CTV) message from m to n.

The following is an expression for the decoding procedure of the proposed algorithm for layered scheduling of 5G NR LDPC codes.

Step 1: Initialization.

For every VN n, AP information update $\tilde{\gamma}_n$ and a priori information γ_n are generated.

$$\tilde{\gamma}_n = \gamma_n = \log\frac{\Pr(x_n = 0|y_n)}{\Pr(x_n = 1|y_n)}$$

The CN messages $\beta_{m,n}$ sent from CN m to VN n are set to zero,

$$\beta_{m,n} = 0$$

Step 2: VN processing. The VTC message from n to m $\alpha_{m,n}$ is calculated by:

$$\alpha_{m,n} = \tilde{\gamma}_n - \eta . \beta_{m,n}$$

Step 3: CN processing. The CTV messages $\beta_{m,n}$ are computed by:

$$\beta_{m,n} = A. \begin{cases} \min 2' & \text{if } |\alpha_{m,n}| = \min 1 \\ \min 1' & \text{if } |\alpha_{m,n}| \neq \min 1 \end{cases}$$

where

$$A = \prod_{n' \in H(m)\backslash n} \operatorname{sign}(\alpha_{m,n'}).$$

$$\Delta\min = \min 2 - \min 1$$

$$\min 1' = \delta . \min 1$$

$$\min 2' = \begin{cases} \Delta\min & \text{if } \Delta\min > \delta . \min 1 \\ \Delta\min + \delta . \min 1 & \text{if } \Delta\min \leq \delta . \min 1 \end{cases}$$

The $\text{index\_}\min 1$ indicates the position of $\min 1$, and the first two minimum values among all VTC input messages are $\min 1$ and $\min 2$. If there are several min1 values, the $\text{index\_}\min 1$ will be considered the smallest index. The VN n' belongs to the set of $H(m)$, excluding VN n itself.

Step 4: A posteriori information is given by:

$$\tilde{\gamma}_n = \alpha_{m,n} + \beta_{m,n}$$

In this project, the decoder terminates once it reaches the maximum number of iterations.

The optimal coefficients (δ, η) are found using a simulation program or the Density Evolution method. In this paper, for 5G LDPC codes, the optimal values (δ, η) are obtained by 0.75 and 0.875, respectively.

3. Results and Discussion

The decoding performance of the various algorithms was evaluated through Monte Carlo simulations for codeword lengths of 8832 and 6720, and code rates of $1/2$ and $2/3$, based on BG1 of 5G LDPC codes. The simulations assume an AWGN channel and BPSK modulation. In this setup, messages $\alpha_{m,n}; \tilde{\gamma}_n; \gamma_n$ are quantized to 6 bits, while $\beta_{m,n}$ uses 4 bits. A maximum of 20 iterations was established. The algorithms simulated were the NMS (the normalization factor $\alpha = 0.75$) (Declercq et al. 2014), Improved OMS (IOMS) (the offset factors $\gamma = 0.875$ and $\eta = 0.5$) (Tran-Thi et al. 2021), MS, Simplified 2-Dimensional Scaled (S2DS) Min-Sum (the normalization factor is 0.75) (Cho et al. 2017) and the proposed DMNMS.

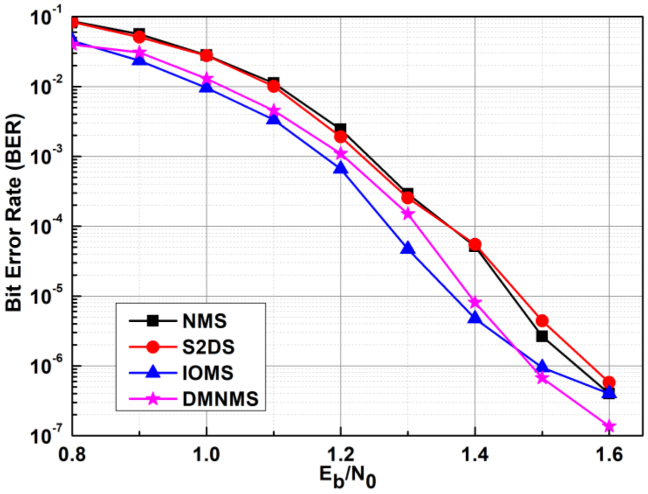

Figure 1: BER performance for the codeword length of 8832 and code rate $1/2$ of different LDPC decoders

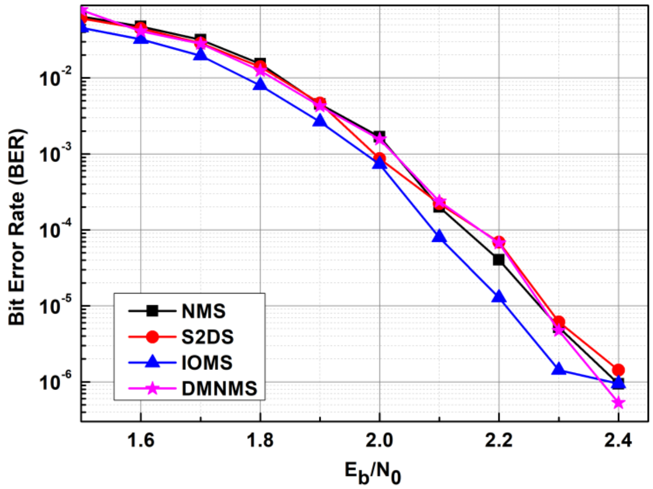

Figure 2: BER performance for the codeword length of 6720 and code rate $2/3$ of different LDPC decoders

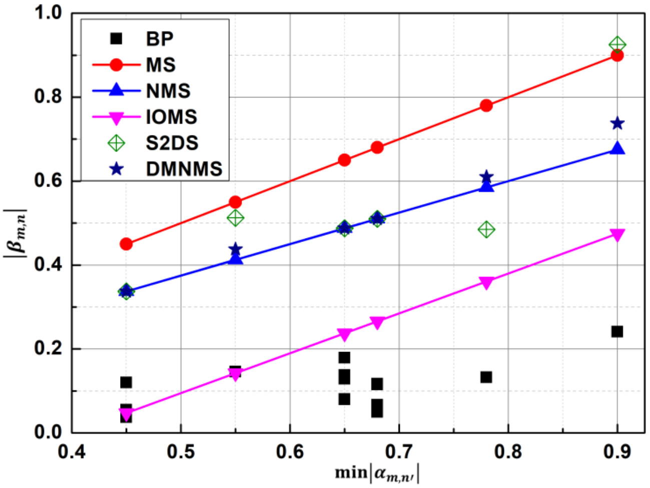

Figure 3: Check node information distribution with $d_c = 6$ for different algorithms

Figures 1 and 2 show the simulated Bit Error Rate (BER) curves. It is evident that at a BER of 10^{-6}, the proposed DMNMS algorithm achieves a performance gain of up to 0.07 dB over the S2DS algorithm, whereas the IOMS algorithm starts to exhibit an error floor. The error-correcting capabilities of the S2DS and NMS algorithms are almost the same. In 5G LDPC code, low-degree VNs are typically more prone to errors. Compared to other VN degrees, the error probability is 10^3 times higher on the degree-1 VNs of the expansion portion of the 5G LDPC codes (Le Trung et al. 2019). For this reason, the suggested approach enhances the decoding performance by applying two correction factors to both the VN and CN processes.

Figure 3 depicts the magnitude of the CN output information, which depends on the magnitude of the first minimum among the input information values associated with a check node. As shown in Fig. 3, the CTV messages from the suggested DMNMS approach are contrasted with those of some current MS-based decoding algorithms, including NMS, S2DS, and IOMS, as well as MS and BP methods, to comprehend the suggested concept. Six samples were tested. The output CTV information produced by the DMNMS algorithm closely resembles that of the NMS algorithm, but differs significantly from the S2DS algorithm. To enhance decoding efficiency and reduce the hardware complexity of the CN process, a correction factor is incorporated into the VN process, as described in (3).

To minimize memory usage, the output CTV messages are stored in a compressed format (Declercq *et al.* 2014). Thus, instead of sending $\{\text{signs}, \min 1, \min 2, \text{index\_} \min 1\}$, a CTV message of the DMNMS algorithm consists of $\{\text{signs}, \min 1, \Delta\min, \text{index\_} \min 1\}$, which lowers the memory consumption and the connectivity complexity of the LDPC decoder. The magnitudes of $\min 1$, $\min 2$ and $\Delta\min$ have been compared in an analysis to demonstrate the validity of the aforementioned assumption. Fig. 4, which summarizes the analytical results, demonstrates that a smaller quantization bit width is required since the magnitude of $\Delta\min$ is smaller than that of $\min 2$.

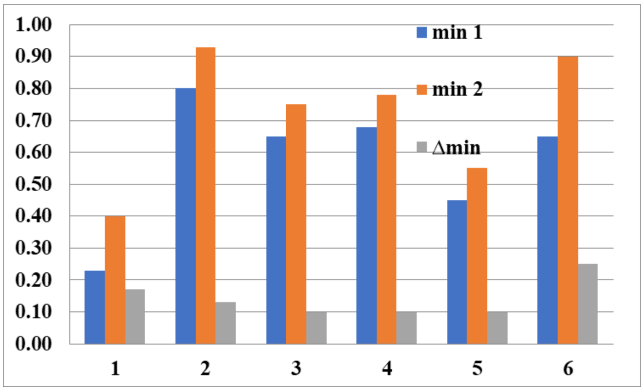

Figure 4: The average magnitudes of $\min 1$, $\min 2$, and $\Delta\min$

4. Conclusion

The DMNMS decoding algorithm for 5G NR LDPC codes is presented, further refining the NMS algorithm for layered scheduling. Compared to the S2DS and NMS, the proposed algorithm enhanced decoding performance and decreased check node memory usage.

The proposed algorithm adjusted both the check node and the variable node processing using normalized factors (δ, η). To minimize memory usage, it calculated the difference between the first two minima among VTC messages. Simulation results showed that this algorithm achieved coding gains of up to 0.07 dB compared to other MS-based decoding algorithms.

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